A Row-Action Based L_1-Minimization Approach to Robust Fluorescent Tomography

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Abstract-We present a row-action method based on minimization of the L_1 norm for improving the accuracy of fluorescent tomography in reconstruction of fluorescent objects. The method is validated using a CW system and milk-based phantoms.

***Index Terms*—**Medical imaging, fluorescent tomography, optical tomography, molecular imaging, in-vivo imaging

Fluorescent tomography (FT) is a non-invasive optical imaging technique for depth-resolved imaging of fluorescent-tagged objects located up to few centimeters under the skin and has found applications in early cancer detection and drug development and monitoring [1]. In this methodology, the medium is pumped with laser light at the excitation wavelength of the fluorophores and the intensity of fluorescent emission is measured at different locations on the boundary of the medium. By applying the finite element method (FEM) formulation of the diffusion approximation, a linear model can be derived for fluorescent tomography as follows

$$M = \mathbf{Z}X\,, \tag{1}$$

where the fluorophore concentrations on the mesh are represented by the $K \times 1$ vector X (where K is the number of FEM mesh nodes), M is the $N_s N_d \times 1$ measurement vector (where N_s and N_d are source and detector numbers respectively) and $\mathbf{Z}$ is the system matrix [3].

The linear problem of Eq. (1) is highly ill-conditioned. Inverse solvers, therefore, resort to regularization schemes. Furthermore, it is often of interest to characterize small fluorescent objects which form in tissue when neoplastic lesions are tagged with fluorescent probes. In [3] an optimization approach for fluorescent object localization is proposed which utilizes the sparse distribution of fluorescent objects to improve reconstruction accuracy, where it is shown it is possible to accurately localize and characterize fluorescent objects. Solving for the fluorophore distribution using the sparsity of the solution results in an optimization problem which involves the minimization of the L_1 norm of the solution (defined as the sum of the absolute values of the vector elements) [4]. This problem is convex, as all optimization terms and constraints are convex. While the problem is solvable using convex optimization [5], for in-vivo imaging of large tissues this approach becomes increasingly numerically demanding as the measurement size grows significantly.

A solution that has been used in fluorescent, and other areas of medical imaging, for solving linear problems of large sizes is the application of row-action type methods, such as the algebraic reconstruction technique (ART) [6].

In this paper we propose a modification to ART that seeks to minimize the L_1 norm of the solution in the iterations. Other row-action iterative methods for the purpose of solving a linear equation while minimizing the solution's L_1 norm have been developed; however, we have found these methods to be highly sensitive to the parameter settings for our tomographic applications [7].

Denoting the transpose operator by superscript ^{T} and the p^{th} row of matrix $\mathbf{Z}$ by vector $\mathbf{Z}_p$, at any given iteration ART seeks to update the current solution $x^{(i)}$ to the new solution $x^{(i+1)}$ by solving the following optimization problem

$$x^{(i+1)} = \arg\min_x \left\| x - x^{(i)} \right\| \text{ subject to } \mathbf{Z}_p x = M(p), \tag{2}$$

which results in the solution

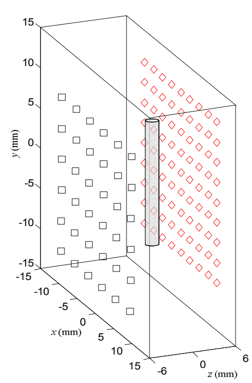
$$x^{(i+1)} = x^{(i)} - \left(M(p) - \mathbf{Z}_p x^{(i)}\right)\frac{\mathbf{Z}_p}{\mathbf{Z}_p^T\mathbf{Z}_p}. \tag{3}$$

We force sparsity by minimizing the L_1 norm of the new solution vector $x^{(i+1)}$ in the cost function of the optimization problem of Eq. (2) as follows

$$x^{(i+1)} = \operatorname*{argmin}_x \left\| x - x^{(i)} \right\| + \lambda \left\| x \right\|_1 \text{ subject to} \mathbf{Z}_p x = M(p), \tag{4}$$

Using Lagrange multipliers we obtain the following solution for $x^{(i+1)}$

$$x^{(i+1)} = x^{(i)} - \left(M(p) - \mathbf{Z}_p x^{(i)}\right)\frac{\mathbf{Z}_p}{\mathbf{Z}_p^T\mathbf{Z}_p} - \lambda\left(\left\|\mathbf{Z}_p\right\|_1 \frac{\mathbf{Z}_p}{\mathbf{Z}_p^T\mathbf{Z}_p} - \mathbf{1}\right), \tag{5}$$



i. Set $i = 1$, $x^{(1)} = x_0$ (initial guess)
ii. Iterate the following loop K times
iii. For $p = 1$ to $N_s \times N_d$

$$x^{(i+1)} = x^{(i)} - \left(M(p) - \mathbf{Z}_p x^{(i)}\right)\frac{\mathbf{Z}_p}{\mathbf{Z}_p^T\mathbf{Z}_p} - \lambda\left(\left\|\mathbf{Z}_p\right\|_1 \frac{\mathbf{Z}_p}{\mathbf{Z}_p^T\mathbf{Z}_p} - \mathbf{1}\right)$$

$$x^{(i+1)} = \max\left(x^{(i+1)}, 0\right)$$

End

Fig. 1 Left: the measurement geometry. The box depicts a portion of the phantom and the square and diamonds depict the source and detectors, respectively. The fluorescent tube is shown by the gray rod. Right: The proposed iterative method.

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where **1** is the all one vector of size $K \times 1$ and the parameter λ is adjusted empirically. The proposed iterative method is presented in Fig. 1 in detail.

We have used a non-contact continuous-wave system for the validation of the proposed method. A He-Ne laser produces CW light at a wavelength of 632 nm is used to pump a milk-based slab phantom at 36 source locations (as shown in Fig. 1) and the light at excitation and emission wavelengths are imaged by a cooled CCD camera. A DMSO-based solution of Oxazine 750 Perchloratel dye from Exciton, Inc. (excited at 633 nm, emission at 700 nm) is used to fill a glass tube with a diameter of 2 mm. The fluorescent tube is imaged at three different depths, z = 3 mm, 0 mm and -2 mm (i.e. 3 mm, 6 mm and 8 mm from the front surface of the tissue phantom that faces the CCD camera). The reconstructions are performed using ART and the proposed method as illustrated in Fig. 2. The reconstructions are performed under the same conditions, i.e. number of iterations (200) and data thresholds, for the two algorithms. As seen, the reconstructions using ART shown in Fig. 2 (b), (d), and (f) tend to be spread along depth, i.e. the z axis, and, hence, have poor depth accuracy while the images reconstructed using the proposed method, on the other hand, are better localized around the correct corresponding depths.

In conclusion, we proposed a row-action based method, as a modification of ART, based on minimization of the L_1 norm of the fluorophore distribution. The proposed method increases the accuracy of the fluorescent tomography in reconstruction of fluorescent object as validated experimentally by imaging fluorescent tubes inserted at different depths in a milk-based tissue phantom. While minimizing of the L_1 norm of the solution results in sparse solutions, the proposed method has been capable of reconstruction objects that are non-sparse in one or more dimensions, such as the fluorescent tubes used in the experiments discussed here.

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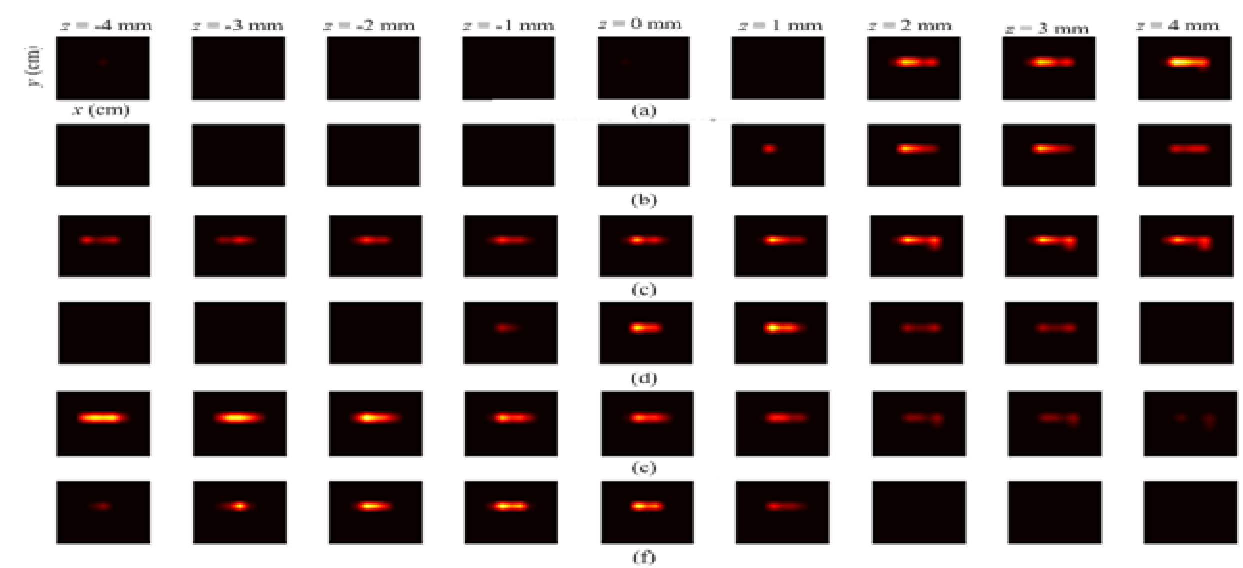

Figure 2 The results of the reconstruction the tubes using ART, in (a), (c) and (e), versus the proposed method, in (b), (d) and (f). The first (a, b), second (c, d) and third (e, f) two rows correspond to the case where the fluorescent tube is inserted at z = 3 mm, 0 mm and -2 mm, respectively. As seen, the reconstructions using the proposed method are more accurately localized in the z direction with respective to the actual tube location than the reconstructions obtained using ART. The reconstructions are performed under the same conditions.